



## CENGAGE | WEBASSIGN

### Application of Calculus(Integration)

**Q1. Verify that F is an antiderivative of f.**

$$F(x)=(x^3)/3+5x^2-x+9, f(x)=x^2+10x-1$$

**Ans.** F(x) is an anti derivative of f(x) because  $F'(x)=f(x)$  for all x.

**Q2. Consider the following function.**

$$G(x)=6x; f(x)=6$$

**Ans.** a) verify that G is an antiderivative of f.

**Ans.** G(x) is an antiderivative of f(x) because  $G'(x)=f(x)$  for all x.

b) Find all antiderivative of f.(Use C for constant of integration.)

**Ans.**  $6x+C$ . (Since  $\int a dx=ax+C$ , where a is any constant and C is constant of integration)

**Q3. Find the indefinite integral.(Use C for constant of integration.)**

$$\int x^a dx.$$

**Ans.**  $(1/(a+1))x^{a+1}+C$ . (Since  $\int x^a dx=(x^{a+1})/(a+1)+C$ , where a is any constant)

**Q4. Find the indefinite integral.(Use C for constant of integration.)**

$$\int (7+u+u^2) du$$

**Ans.**

$$\int (7+u+u^2) du \\ \Rightarrow \int 7 du + \int u du + \int u^2 du \Rightarrow 7u+u^2/2+u^3/3+C. \\ \text{(since } \int x^a dx = (x^{a+1})/(a+1)+C \text{)}$$

**Q5. Find the indefinite integral.(Use C for constant of integration.)**

$$\int (x^7-1)/x^3 dx.$$

**Ans.**

$$\int (x^7-1)/x^3 dx \Rightarrow \int (x^4-(1/x^3)) dx \\ \Rightarrow \int x^4 dx - \int x^{-3} dx \\ \Rightarrow x^5/5 - (x^{-3+1})/(-3+1)+C$$

$$\Rightarrow x^5/5 + 1/2x^2 + C. \quad (\text{Since } \int x^a dx = (x^{(a+1)})/(a+1) + C; \int x^a dx = (x^{(a+1)})/(a+1) + C)$$

**Q6. Find the indefinite integral. (Use C for constant of integration.)**

$$\int (x^4 - 9x^2 + 4)/x^2 dx.$$

**Ans.**

$$\Rightarrow \int (x^4 - 9x^2 + 4)/x^2 dx$$

$$\Rightarrow \int (x^4/x^2 - 9x^2/x^2 + 4/x^2) dx$$

$$\Rightarrow \int (x^2 - 9 + 4x^{-2}) dx$$

$$\Rightarrow \int x^2 dx - 9 \int dx + 4 \int x^{-2} dx$$

$$\Rightarrow x^3/3 - 9x - 4/x + C. \quad (\text{Since } \int x^a dx = (x^{(a+1)})/(a+1) + C; \int a dx = ax + C, a \text{ is any constant})$$

**Q7. Find the indefinite integral. (Use C for constant of integration.)**

$$\int x \sqrt{x^4 + 5} dx.$$

$$\text{Ans. } 2/11x^{11/2} + 2/5x^{5/2} - 10/3x^{3/2} + C.$$

**Q8. Find f by solving the initial value problem.**

$$f'(x) = 9\sqrt{x} \quad f(9) = 63$$

**Ans.**

$$\text{Here, } f'(x) = 9\sqrt{x}$$

$$\text{If } f'(x) = g(x) \text{ then } f(x) = \int g(x) dx$$

$$\Rightarrow f(x) = \int 9\sqrt{x} dx = \int 9x^{1/2} dx$$

$$\Rightarrow f(x) = 9 \cdot x^{1/2+1} / (1/2+1) + C$$

$$\Rightarrow f(x) = 9 \cdot x^{3/2} / (3/2) + C$$

$$\Rightarrow f(x) = 9 \cdot 2 \cdot x^{3/2} / 3 + C$$

$$\Rightarrow f(x) = 18 \sqrt{x} + C, \text{ where } C \text{ is integration constant}$$

$$\text{Applying initial value condition : } f(9) = 63$$

$$\Rightarrow 18 \cdot 9 + C = 63$$

$$\Rightarrow C = 63 - 54 = 9$$

$$\text{Thus, } f(x) = 18\sqrt{x} + 9$$

**Q9. Find the function f given that the slope of the tangent line to the graph of f at any point (x, f(x)) is f'(x) and that the graph of f passes through the given point. (Remember to use absolute value whenever required.)**

$$f'(x) = 6x^2 - 10x + 8; \quad (2, 16)$$

$$\text{Ans. Here, } f'(x) = 6x^2 - 10x + 8$$

$$\text{If } f'(x) = g(x) \text{ then } f(x) = \int g(x) dx$$

$$\Rightarrow f(x) = \int (6x^2 - 10x + 8) dx$$

$$\Rightarrow f(x) = 2x^3 - 5x^2 + 8x + C, \text{ where } C \text{ is integration constant}$$

$$(\text{Since } \int x^a dx = (x^{(a+1)})/(a+1) + C; \int x^a dx = (x^{(a+1)})/(a+1) + C)$$

$$\text{Applying initial value condition : } f(2) = 16$$

$$\Rightarrow 2 \cdot 2^3 - 5 \cdot 2^2 + 8 \cdot 2 + C = 16$$

$$\Rightarrow C = 4$$

$$\text{Thus, } f(x) = 2x^3 - 5x^2 + 8x + 4.$$

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**Q10.** Find the function  $f$  given that the slope of the tangent line to the graph of  $f$  at any point  $(x, f(x))$  is  $f'(x)$  and that the graph of  $f$  passes through the given point.  
(Remember to use the absolute value where appropriate.)

$$f'(x) = 7/x + 1; (1, 9)$$

**Ans.** Here,  $f'(x) = 7/x + 1$

If  $f'(x) = g(x)$  then  $f(x) = \int g(x) dx$

$$\Rightarrow f(x) = \int (7/x + 1) dx$$

$\Rightarrow f(x) = x + 7 \ln|x| + C$ , where  $C$  is integration constant

Applying initial value condition,  $f(1) = 9$ , we get,

$$\Rightarrow 1 + 7 \ln|1| + C = 9$$

$$\Rightarrow C = 8 \quad (\text{Since } \ln|1| = 0)$$

Thus,  $f(x) = x + 7 \ln|x| + 8$

**Q11.** Find the area (in square unit) of the region under the graph of the function  $f$  on the interval  $[-7, 1]$ , using the fundamental theorem of calculus;  $f(x) = 3$ .

**Ans.**

$$\text{Area} = \int_{-7}^1 3 dx$$

$$\Rightarrow 3x$$

$$\Rightarrow 24 \text{ square unit.}$$

**Q12.** Find the area (in square unit) of the region under the graph of the function  $f$  on the interval  $[-1, 5]$ ;  $f(x) = 2x + 8$ .

**Ans.**

$$\text{Area} = \int_{-1}^5 (2x + 8) dx$$

$$= 72 \text{ square unit.}$$

**Q13.** A study proposed in 1980 by researchers from major producers and consumers of world's coal concluded that coal could and must play an important role in fueling global economic growth over the next 20 years. Suppose the world production of coal in 1980 was 7.5 billion metric tons. If output increased at the rate of  $7.5e^{(0.05t)}$  billion metric tons/year in year  $t$  ( $t=0$  corresponding to 1980), determine how much coal was produced worldwide between 1980 and the end of 20th century.

(Round your answer to one decimal place.)

**Ans.**

$$\text{Here, } r(t) = 7.5e^{(0.05t)}$$

The total coal produced in between 1980 and end of 20th century is given by

$$\int_0^{20} r(t) dt$$

$$\begin{aligned}
 &= \int_0^{20} 7.5e^{0.05t} dt \\
 &= 7.5 \int_0^{20} e^{0.05t} dt \\
 &= 7.5 [e^{0.05 \cdot 20} - e^{0.05 \cdot 0}] / 0.05 = 150 [e^1 - e^0] = 150 [e - 1] \\
 &= 257.7 \text{ billion metric tons.}
 \end{aligned}$$

**Q14. Seat belt use** Suppose the percentage of drivers using seat belts from 2001 through 2009 is modeled by the function  $f(t) = 73.8(t+1)^{0.059}$ , where  $t$  is measured in years with  $t=0$  corresponding to the beginning of the year 2001. What was the average percentage use of this device over the period from beginning of 2001 through the end of 2009 ( $t=9$ )? (round your answer to 1 decimal place).

**Ans.**

Here,  $f(t) = 73.8(t+1)^{0.059}$  ;  $0 \leq t \leq 9$

$$\text{Average percentage} = \frac{1}{(b-a)} \int_a^b f(t) dt$$

$$\Rightarrow \text{Average percentage} = \frac{1}{(9-0)} \int_0^9 73.8(t+1)^{0.059} dt$$

$$= 73.8/9 \int_0^9 (t+1)^{0.059} dt$$

$$= 8.2 [(t+1)^{0.059+1}] / 0.059 + 1 \Big|_0^9 = (t+1)^{0.059+1} / 0.059$$

$$= 8.2/1.059 [11.46 - 1]$$

$$= 7.743 \cdot 10.46$$

$$= 80.99\%$$

**Q15. A car moves along a straight road in such a way that its velocity (in feet per second) at any time  $t$  (in seconds) is given by  $v(t) = 3t(9-t^2)$ , ( $0 \leq t \leq 3$ ). Find the distance travelled by the car in 3 seconds from  $t=0$  to  $t=3$ .**

**Ans.**

Here,  $v(t) = 3t\sqrt{9-t^2}$ ,  $0 \leq t \leq 3$

The distance travelled by the car in 3 seconds is given by,

$$s(t) = \int_0^3 3t\sqrt{9-t^2} dt$$

$$= 3 \int_0^3 t\sqrt{9-t^2} dt$$

Apply u-substitution, i.e; let  $u = 9-t^2$ , then we have,

$$s(t) = 3 \int_a^b -\sqrt{u}/2 du$$

$$= 3 \cdot (-) \int_a^b \sqrt{u}/2 du \quad (\text{since } \int_a^b x dx = - \int_b^a x dx)$$

$$\begin{aligned} &= 3^2 \left( -\frac{1}{2} \int_0^9 \sqrt{u} \, du \right) \\ &= 3^2 \left[ \frac{9^{3/2+1}}{(1/2+1)} - \frac{0^{3/2+1}}{(1/2+1)} \right] \\ &= 3^2 \left[ \frac{9^{5/2}}{3/2} - 0 \right] \\ &= 3^2 \left[ \frac{2}{3} \cdot 3^5 \right] \\ &= 3^2 \cdot 18 \\ &= 27 \end{aligned}$$

Hence the required distance is 27 ft.





